Phys 1010

Lecture 2

Mechanical Properties of Matters (Metals)

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Contents

- Concepts of stress and strain.
- Elastic deformation.
- Plastic deformation.
- Types of Elasticity modulus.

1- Young's (Tensile) Modulus

2- Shear (Rigidity) Modulus

3- Bulk (Volume) Modulus

Stress and Strain

- Stress is applied force per unit area.

$$\text{stress}(\sigma) = \frac{\text{force}}{\text{area}} = \frac{F}{A} \qquad (\text{N/m}^2)$$

- Strain is ratio of deformation to original length.

$$\text{strain}(\varepsilon) = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L_0} \qquad (???)$$

Elastic modulus (E)

- **Elastic modulus is the proportionality constant.**

Stress = E Strain

$$\frac{F}{A} = E\,\frac{\Delta L}{L_o}$$

E = Stress/Strain

= (F/A)/(ΔL/L) (N/m^2)

Materials Deformation

Elastic Materials

A material is called elastic if the deformation produced in the body is completely recovered after the removal the load

i.e. Deformation is reversible

Palastic Materials

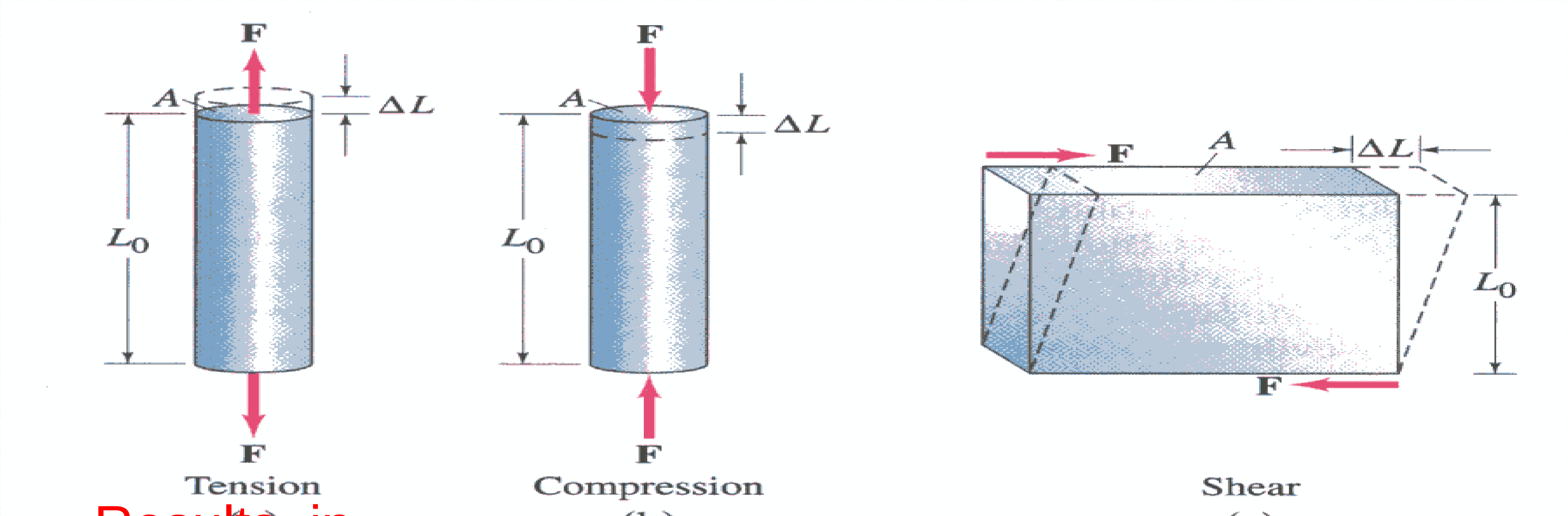
A material is called elastic if the deformation produced in the body is not completely recovered after the removal the load

i.e. Deformation is irreversible

Stress: It is the instantaneous perpendicular force (F) per unit cross - sectional area (A_o). .i.e. is related to the force causing the deformation

$$\sigma = \frac{F}{A} \quad N/m^2 \quad \text{or} \quad Ib/in^2$$

- **Material stress takes three forms:**



Results in elongation

Results in contraction

Results in deformation

Strain (ε)

Is a measure of the degree of deformation?
and is defind as
It is the ratio between the change in length (ΔL) and the original length (L_o).

$$\varepsilon = \frac{\Delta L}{L_o}$$ ε

Strain

$$\overbrace{\hspace{10em}}$$

Elastic Deformation

Palastic Deformation

Elastic Modulus

Young's (Tensile) modulus

Shear (Rigidity) modulus

Bulk (Volume) modulus

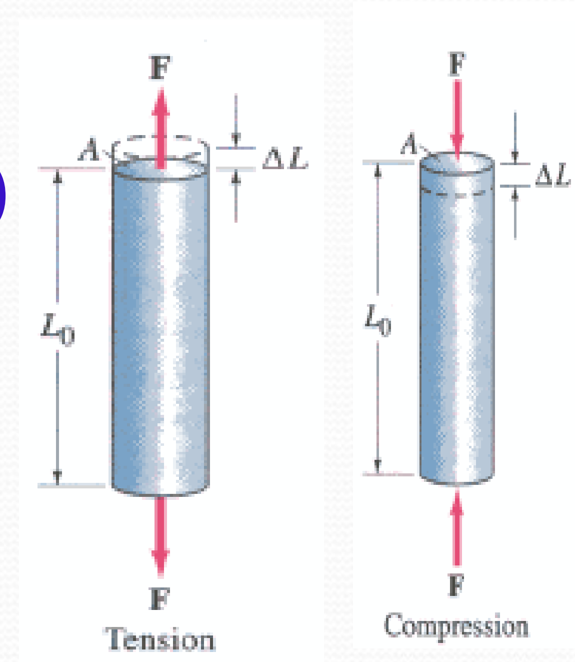
Young's Modulus (Y)

- It is the ratio between the stress and the strain

$$\text{stress}(\sigma) = \frac{\text{force}}{\text{area}} = \frac{F}{A} \quad (\text{N/m}^2)$$

$$\text{strain}(\varepsilon) = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L_0} \quad (???)$$

$$\text{Y} = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta \text{L}/\text{L}} \quad (\text{N/m}^2)$$



Examples

- **Ex. 1: A mass of 80 Kg is hung on a steel wire having 18m long and 3m diameter. What is the elongation of the wire, knowing Young's modulus for steel is**

$$A = \pi \times (1.5)^2$$

$$\sigma = F/A = 80 \times 9.8 \ / A;$$

$$\varepsilon = \Delta L/L = \Delta L/ 18$$

$$Y = \sigma / \varepsilon$$

$$21 \times 10^{10} \ N/m^2$$

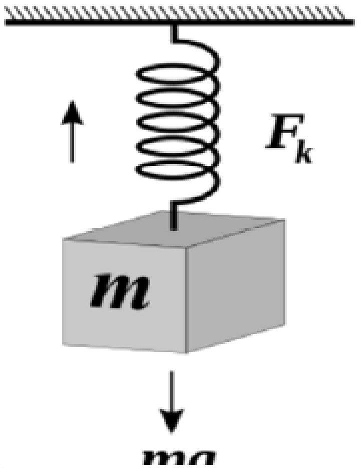
Examples

- **Ex. 2**: A structured steel rod has a radius R of 9.5 mm and a length L of 81 cm. A force F 6.2×10^{4} N
- stretches it axially. $\left(E_{steel} = 2\times10^{11}\ \text{N/m}^2\right)$
- (a) What is the stress in the rod?
- (b) What is the strain?
- (c) What is the elongation of the rod under this rod?

Hooke's Law

In mechanics and Physics, Hooke's law of elasticity is an approximation that states that the extension of a spring is in direct proportion with the load applied to it. Many materials obey this law as long as the load does not exceed the material's elastic limit. Materials for which Hooke's law is a useful approximation are known as inelastic limit or "Hookean" materials.

Hooke's law in simple terms says that strain is directly proportional to Stress. Mathematically, Hooke's law states that $F = -k\,x$

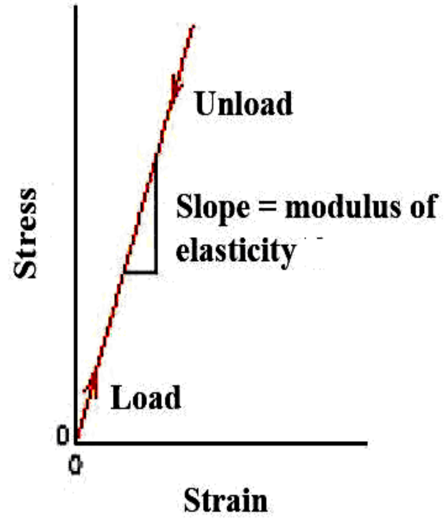


where

X is the displacement of the spring's end from its equilibrium position

F is the restoring force exerted by the the spring on that end

k is a constant called the *rate* or *spring constant*) in SI units: N/m



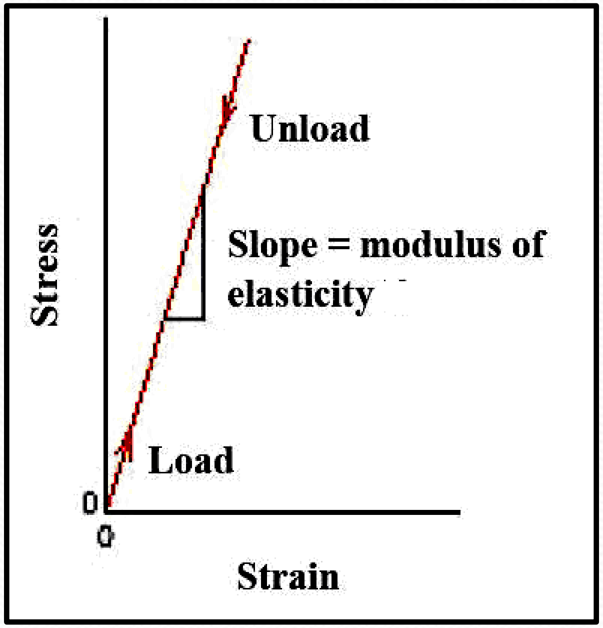
We may view a rod of any elastic material as a linear spring. The rod has length L and cross-sectional area A. Its extension (strain) is linearly proportional to its tensile stress σ, by a constant factor, the inverse of its modulus of elasticity, Y, hence

- **If a rod is stretched by a force F distance ΔL, then**

$$Y = \frac{F_1 L}{A \Delta L} \Rightarrow F = \frac{YA}{L} \Delta L$$

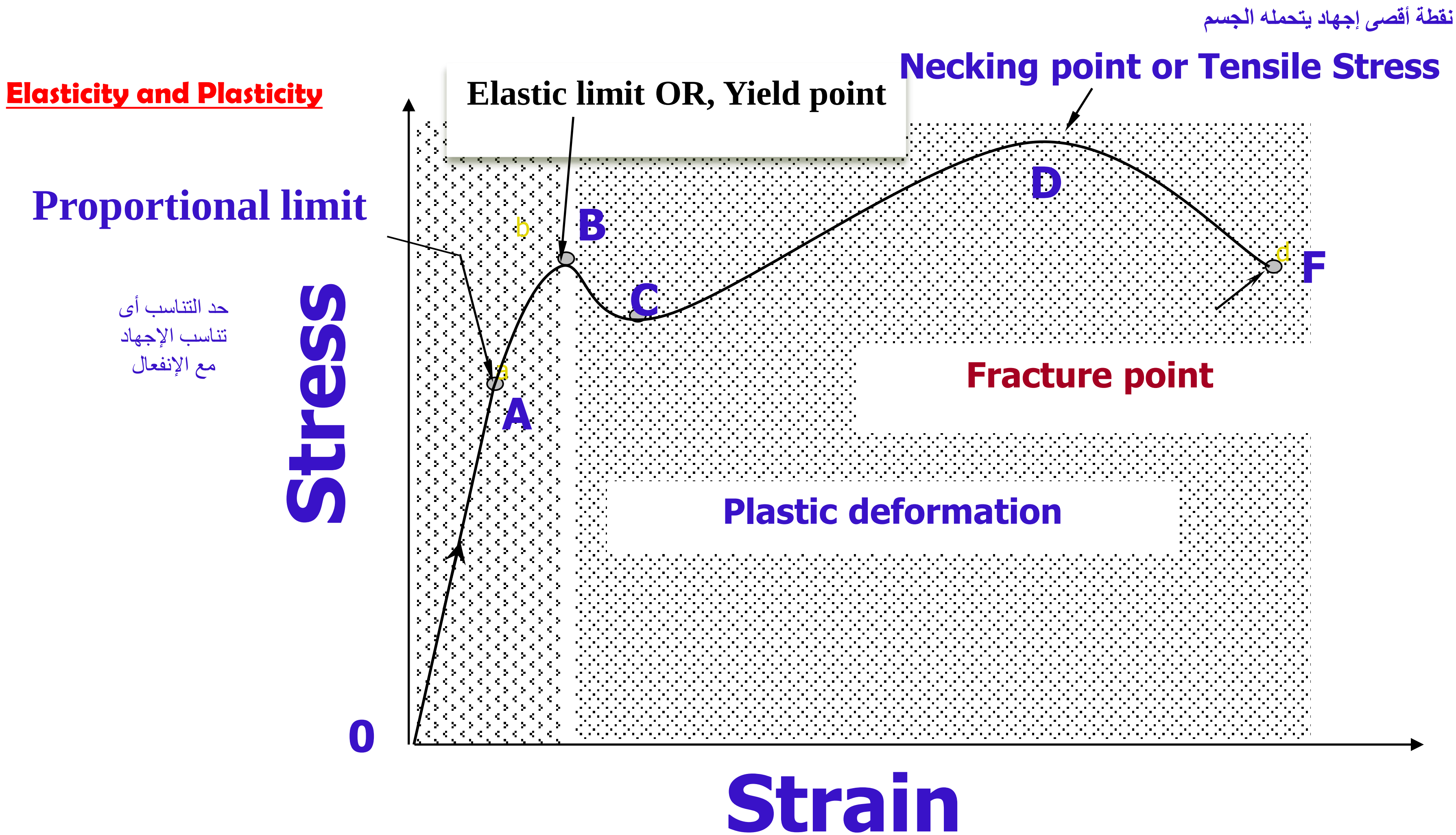
$$\text{Let } K = \frac{YA}{L} \quad \text{(Constant Force)}$$

$$\Rightarrow F = K\,\Delta L \quad = \mathrm{K}\,\Delta \mathrm{x}$$

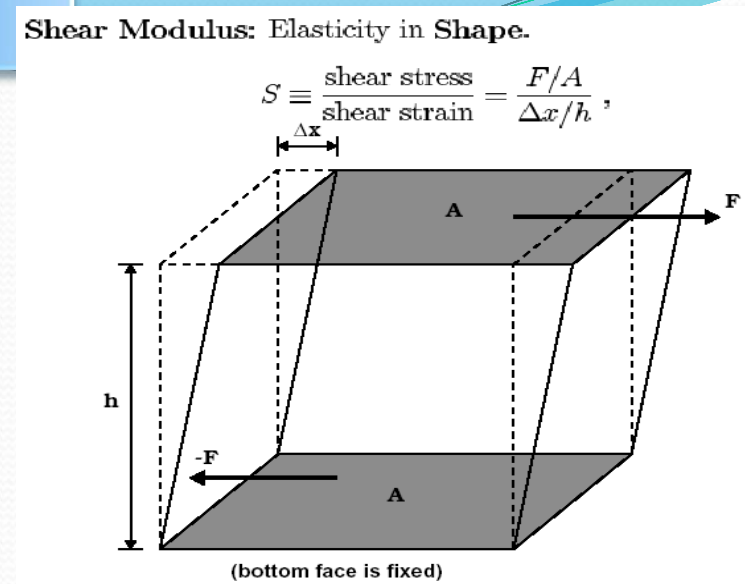


Stress – Strain Behavior

Elasticity and Plasticity



Shear Modulus (Elasticity in Shape)



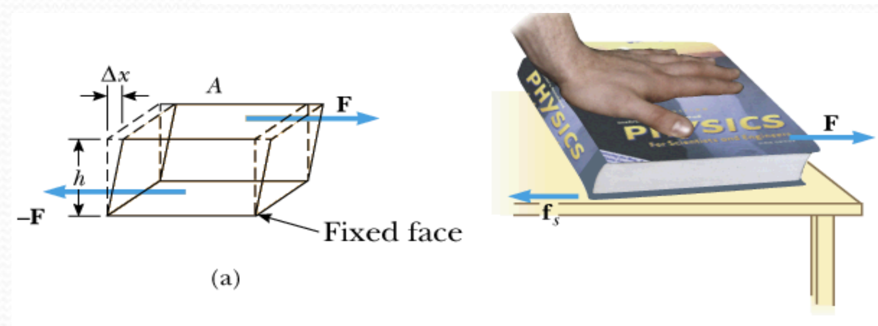
Shear Stress

$$\text{Shear Stress} = \frac{F_t}{A} \qquad N/m^2 \qquad \text{or} \qquad Ib/in^2$$

Shear Strain

$$\text{Shear Strain} = \frac{\Delta x}{h}$$

tanθ=Δx/h but θ is small so tanθ ≈θ



(a) (b)

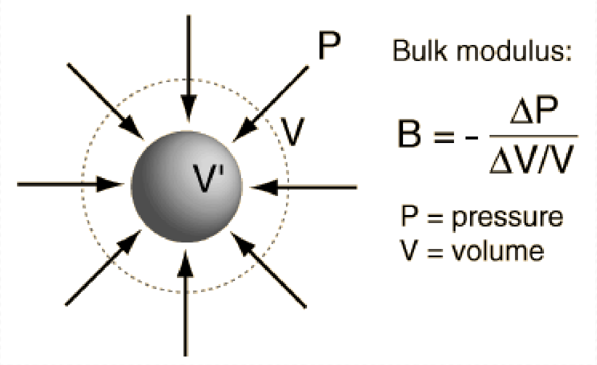
Shear Modulus

$$\text{Shear Modulus (S)} = \frac{\text{Shear Stress}}{\text{Shear Strain}} = (\mathbf{F} / \mathbf{A}) / \theta \qquad N/m^2 \qquad \text{or} \qquad Ib/in^2$$

Bulk Modulus (Elasticity in Volume) (B)

Volume Stress

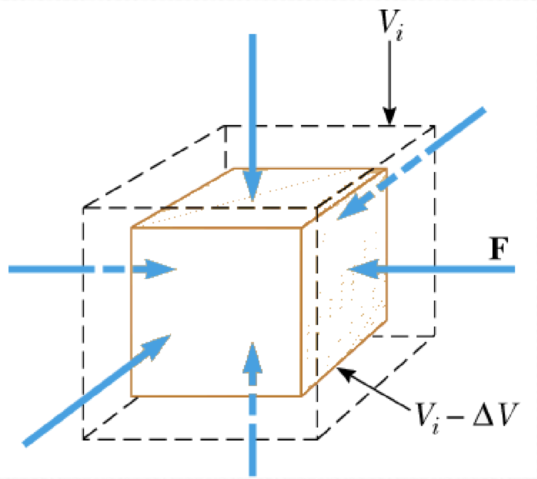
$$\Delta P = \frac{F_n}{A} \qquad N/m^2 \qquad \text{or} \qquad Ib/in^2$$



Volume Strain

$$\text{Volume Strain} = \frac{\Delta V}{V}$$

$$B = -\frac{\text{Volume Stress}}{\text{VolumeStrain}} = -\frac{F/A_o}{\Delta V/V} \qquad N/m^2 \qquad \text{or} \qquad Ib/in^2$$



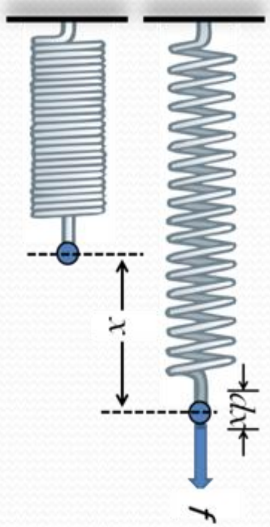
The compressibility factor, $K = \dfrac{1}{B}$

Both solids and liquids have bulk moduli.

Energy Stored in a stretched wire (Elastic potential energy)

- *If a wire streches by extension x by a force Fr, the wire resists this force to restore its original length. This restoring force possesses the wire elastic energy.*
- لو سلك اتشد بقوة يحدث له تمدد فينشأ قوة تسمى بقوة الإسترجاع تعمل على رجوع السلك لشكله الأصلى فيكتسب السلك طاقة مرنة.
- *The restoring force $Fr = - K X$*

k is called the restoring force constant,
The work done is given by:



- *This work done $W_{internal}$ equal potential energy U inside wire*
- **2-6** $$d\,w_{internal} = -F_r \,.\, dx = -k\,(-x)\,.\, dx = kx\,dx$$
- **but** $F_{extenal} = k\,\Delta L$

$$W_{internal} = U = \int_0^{\Delta L} k\,x\;dx = 1/2\,k\,(\Delta L)^2$$

But, $F_{ex} = k\,\Delta L$

$W_{ex} = F_{ex}\,\Delta L$ $W_{ex} = K\,(\Delta L)^2$

- Then $U = \frac{1}{2} F_{ex}\,\Delta L = \frac{1}{2} W_{ex}$ 2-7
- Then $W_{internal} = \frac{1}{2} W_{external}$

- *Multiplying both side by 1/ AL_o*

Then

- **$U/A.L0 = \frac{1}{2} F\,.\Delta L\,(1/A.\,\Delta L0)$ =**
- **$= \frac{1}{2}\ (F/A)\,.\,(\Delta L/L0)$**
- _
- ***Then U= (½ Stress x Strain) V.***

- ***u =U/V is the elatic potential energy per unit volume or elastic energy density.***
- ***U = ½Y . (stress)2 or***
- ***U = Y/2 . (Straim)2***

Poisson's ratio. β

- Is the negative ratio between the lateral strain to longitudinal strain

- = - $(\Delta d/d_0)$ / $(\Delta L/L)_0)$
- ***d_0 is the original diameter and Δ d is the decrease in the diameter***